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A Two-dimensional Maintenance Service Contract Considering Availability and Maintenance Cost

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Abstract. In this paper, we study a two- dimensional Maintenance Service Contract (MSC) characterized by two limits (dimensions) of age and usage. It is considered that an agent offers a two-dimensional MSC by guaranteeing a certain level of equipment available to consumers. The agent needs to reduce the total maintenance cost to offer competitive MSC prices. Preventive maintenance actions (PM) are periodically carried out, and each PM action is considered to improve reliability modeled by reducing the failure rate function. Two decision variables (the PM interval (T) and the reduction in the intensity function (δ)) are obtained by considering two performance measures that are relevant to agents and consumers (i.e., availability and total maintenance cost). A numerical example is presented by considering three types of equipment usage rates: low, medium, and high. The optimization of the two performance measures can ensure availability targets and, at the same time, minimize total maintenance costs.

Keywords: Availability; Imperfect preventive maintenance; Maintenance service contract; Total cost; Two-dimensional

1. Introduction

Maintenance Service Contract (MSC) is defined as an equipment maintenance contract that agents offer to consumers within a certain period. Maintenance contracts, generally, are characterized by a time limit (e.g., 1 year), called an MSC with one dimension. However, for equipment such as dump trucks whose failure patterns are affected by age and use, it is necessary to include a usage limit (e.g., 100,000 km) in addition to the age limit. Different types of equipment may have different ways of measuring usage. For example, a photocopier's usage can be measured by the number of copies made, while a machine tool's usage can be measured by the hours it is used. This MSC is referred to as an MSC with two dimensions. To appeal to consumers, the MSC may include promised equipment performance (e.g., Target 94% equipment availability) in addition to the price charged. A comprehensive review of MSC can be found in Murthy and Jack (2014), and MSC is studied

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from the perspectives of consumers (MSC recipients) and agents (MSC providers).

From the consumer side, equipment used to support business processes will deteriorate with age and use, and eventually, failure will occur (Jiang and Murthy, 2008). Maintenance is an effective way to slow down the deterioration of equipment so that failure can be minimized and availability can be kept high (Pariaman et al., 2017; Suthep and Kullawong, 2015; Jackson and Pascual, 2008). Consumers who own equipment, generally need MSC because maintenance activities are not their main business (core business), so they do not need to be done in-house.

Using an agent to maintain equipment through a Maintenance Service Contract (MSC) is more cost-effective and reliable than in-house maintenance, which requires expensive investments in human resources, equipment and technology. MSC offers consumers the benefits of saving on maintenance costs and improving equipment performance. Many studies have explored MSC from the consumer's perspective, including research conducted by Huber and Spinler (2012) and Jensen and Stonecash (2009). In general, research from the consumer's point of view always wants (i) minimum cost or maximum profit, with additional considerations of other performance measures such as (ii) service quality (eg. delivery time) (De-Almeida, 2007) (iii) product reliability (Laksana and Hartman, 2010), or (iv) availability (Datta and Roy, 2010). Due to the important role of maintenance in maintaining the condition of equipment, the agents respond proactively to the consumers' needs by offering MSCs that promise high availability. In addition to getting regular income, agents can also build good relationships with consumers, which positively impacts the agent's image (brand building).

From the agent's point of view, research was carried out by Tarakci *et al.* (2006), where the agent offers specific incentives based on the combination of target uptime and bonuses in the contract to attract consumers. In addition, reliability improvement is also considered. Generally, the agent looks for the optimal PM time interval to fulfill the performance promise to the customer, which maximizes the profit. MSC can be grouped into two categories: one-dimension MSC and two-dimension MSC. One-dimensional MSC is characterized by a one-time limit (age). For example, the MSC of a piece of equipment is for one year, while the two-dimension MSC is characterized by an area of a two-dimensional plane where one dimension describes a time limit and the other a usage limit (Iskandar *et al.*, 2014). One-dimensional MSC research has considered several important performance measures for agents (i.e., total cost and benefit) as well as consumers (i.e., reliability and availability). First, many MSC studies consider profit performance measures (Darghouth, Ait-kadi, and Chelbi, 2017; Hamidi, Liao, and Szidarovszky, 2013; Chang and Lin, 2012). Second, the reliability performance measure is considered in addition to the benefits (Laksana and Hartman, 2010). Third, the availability performance measure is also involved (Su and Cheng, 2018; Iskandar *et al.*, 2014). The effect of PM's actions on increasing reliability can be represented through a reduction in (i) failure rate or (ii) virtual age. Some of them are researched by Darghouth, Ait-kadi, and Chelbi (2017) and Pasaribu, Husniah, and Iskandar (2012), who specifically examined one-dimensional MSC with periodic PM policies and the impact of PM reducing a virtual age. Other works are the research of Husniah *et al.* (2019) and Yeh and Chang (2007), which examine one-dimensional MSC with a periodic PM policy model using an intensity reduction function. There is also research by Iskandar and Husniah (2017) and Yeh, Kao, and Chang (2009), who investigate one-dimensional contracts using a PM policy that ensures equipment reliability.

Meanwhile, two-dimensional MSC studies have not received much attention. As in one-dimensional MSC, the most commonly used performance measure in two-dimensional MSC is profit (Huang, Gau, and Ho, 2015; Husniah *et al.*, 2014). No studies have considered the availability of two-dimensional MSCs. At the same time, consumers want high equipment availability to be guaranteed. The motivation to prioritize availability is because the losses (costs) due to equipment downtime are very large. For example, the breakdown of equipment such as draglines (in the mining business) and airplanes (in the transportation business) will result in huge losses for the company operating the equipment. Performance in two-dimensional MSC can be achieved by implementing various preventive maintenance (PM) policies, which can be divided into two categories, namely periodic and non-periodic (Jiang and Murthy, 2008).

In this paper, we propose a two-dimensional MSC that implements the imperfect PM policy carried out periodically. The effect of the PM action on the two-dimensional MSC will be developed by extending the formulation of the one-dimensional MSC. As in the one-dimensional MSC studied by Yeh and Chang (2007), It is assumed that the effect of PM reduces the intensity function. Furthermore, this paper focuses on two-dimensional MSC, The study is carried out from the agent's point of view, considering two performance measures, namely: (i) equipment availability and (ii) total maintenance costs, and involving two decision variables, i.e., the PM time interval (T), and the failure rate reduction value (δ) for each PM. As a result, the main contributions of this paper are (i) developing a PM policy that ensures the equipment availability target with minimum costs and (ii) obtaining optimal solutions of PM policies for a two-dimensional MSC with two objective functions, namely maximizing availability and minimizing total cost. This is in accordance with the goals desired by consumers for MSC, which are to ensure high availability (for example, 94%) and simultaneously meet the agency's goal of minimizing total maintenance costs. This paper is organized as follows: The model formulation is given in Section 2, which includes a discussion of the two-dimensional MSC formulation, failure modeling, and PM impact modeling. Section 3 describes an optimization to find the optimal solution that guarantees target availability with the minimum total cost. Section 4 provides numerical examples and a discussion of the results. Finally, we present conclusions and further research topics in Section 5.

2. Model Formulation

This section will detail the two-dimensional MSC under study, failure modeling, and PM effect modeling.

2.1. Two-dimensional MSC

Consider a two-dimensional MSC for equipment (e.g. dump trucks), which is characterized by two parameters, L (time limit) and U (usage limit). For example, a maintenance contract for a dump truck with a time limit of 1 year and a usage limit of 100,000 km. Thus, these two contract limits form a rectangular area (Iskandar *et al.*, 2014). Suppose that the equipment with the rate of use $y = y_0 (= U / L)$ is said to be moderate (normal) usage rate, $y \le y_0$ low usage rate, and high usage rate $y > y_0$. The contract will end at L for a low usage and at U / y for high use. The boundary region of the two-dimensional MSC is given by $[(0, L) \text{x} (0, yU)]$ and $[(0, U / y) \text{x} (0, U)]$ (see Figure 1).

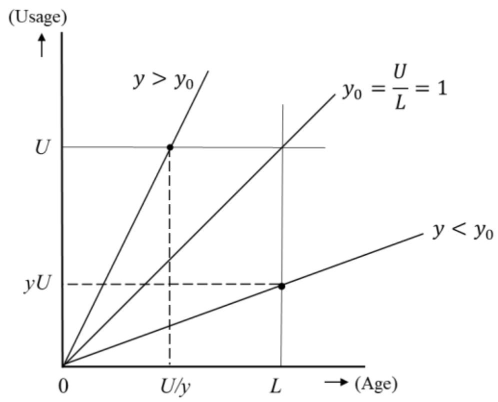

Figure 1 The area of MSC for (a) $y \le y_0$ and (b) $y > y_0$

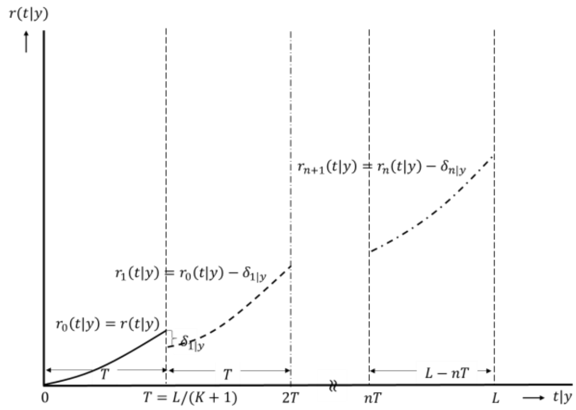

Figure 2 Proposed PM in which each periodic PM action will decrease the failure rate by δ_i

2.2. Failure Modeling

Equipment failures that occur randomly during the contract period are considered random points that fall in the maintenance contract area (which is rectangular). There are three approaches to modeling random points (equipment failure) on a two-dimensional plane (Murthy and Jack, 2014). In this paper, we shall use the formulation of a one-dimensional point process as has been done by Iskandar, Murthy, and Jack (2005). Let Y the random variable represents the usage rate of the equipment (e.g., distance traveled per unit of time (300km/day), machine hours per day). Conditional on $Y = y$, then random points will occur along the time axis (t) so that it can be modeled with a one-dimensional point process. It is assumed that each consumer has a constant use rate and can be different for different consumers.

Equipment is repairable, and the equipment failure over time is modeled with an intensity function (Iskandar and Murthy, 2003). Suppose $r(t \mid y)$ is the conditional failure rate function on $Y = y$. It is assumed that all failures are fixed with minimal repair. Hence, the failure occurrence will follow the Non-Homogeneous Poisson Process (NHPP) with the intensity function $r(t \mid y), t > 0$ given by: $r(t \mid y) = \varphi(t, y)$, where $\varphi(t, y)$ is a non-decreasing function of t and y. Iskandar and Murthy (2003) provide a formulation of the intensity function as a first-order polynomial function. Another formulation that can be used is the Accelerated Failure Time (AFT) (Iskandar *et al.*, 2014), and this formulation will be used in this paper.

Suppose T_y is a random variable representing the time of the first failure for a given usage. The distribution function T_y $F_y(t)$ is given by the Weibull distribution function. Next, the effect of usage patterns on equipment failure modeling will be explained. It is considered that equipment with a high usage rate $(y > y_0)$ will deteriorate faster than equipment with a low usage rate $(y < y_0)$. The AFT formulation can be used to model the effect of this usage pattern as follows. Let $T_0[T_y]$ be the time to first failure for usage rate $y_0[y]$. The relationship of T_0 and T_y is given by, $T_y / T_0 = (y_0 / y)^{\rho}, \rho \ge 1$. If the distribution function for T_0 is given by $F_0(t, \alpha_0)$, with α_0 is the scale parameter, then the distribution function for T_y is the same as the distribution function for T_0 but with the scale parameter given by: $\alpha_y = (y_0 / y)^{\rho} \alpha_0$. Thus, we get $F_y(t, \alpha_y) = F_0((y_0 / y)^{\rho} t, \alpha_y)$. The intensity and cumulative functions with respect to are given by $F_y(t,\alpha_y)$ $r_y(t\,|\,Y=y)=f(t,\alpha_y)/\left(1-F(t,\alpha_y)\right)$ and $R_y(t\,|\,Y=y)=\int_0^t r_y(x)dx$, with $f(t,\alpha_y)$ is a density function with respect to $F(t,\alpha_y)$.

2.3. PM Policy

The agent requires a proper PM policy to reduce failures and downtime, and this will increase availability. In this paper, two PM policies are considered.

2.3.1. PM Policy 1

PM actions are carried out periodically or at the time of $T_i, i=1,2,\ldots,n$. T_i Given by $T_i = L/n+1$ with $T_{n+1}=L$, where L is the contract term. In this policy, the specified value of $T_i, i=1,2,\ldots,n$ is determined to ensure the availability at the interval (T_i,T_{i+1}) meets the defined target and will be described in Section 2.5.

2.3.2. PM Policy 2

PM is done if the failure rate reaches the threshold value θ_y . Suppose $T_i, i=1,2,\ldots,n$ is when done PM is $T_1 \le \ldots \le T_n \le L$. This policy was proposed by Yeh and Chang (2007) to reduce the level of failure rate. So $T_i, i=1,2,\ldots,n$ was determined to minimize the expected number of failures during L.

2.4. PM Effect Modelling

2.4.1. PM Policy 1

As mentioned previously, the effect of PM action can be modeled through (a) virtual age or (b) failure intensity function. Here, the impact of PM is modeled by reducing the failure intensity function and is described as follows. For the use of $Y=y$, the PM action performed at T_i and after PM, the failure rate will be reduced by $\delta_{i|y}$ such that the failure rate in the interval (T_i,T_{i+1}) becomes $r_y(T_i)-\delta_{i|y}, \delta_{i|y}>0$ (See Fig. 2). The decision variable $\delta^*_{i|y}$ depends on y. One can group consumers by usage rate y – e.g., low usage rate moderate usage rate and high usage rate; hence each usage group has a unique $\delta_{i|y}$.

2.4.2. PM Policy 2

In this PM policy, whenever PM (if the failure rate reaches θ_y), failure rate will be reduced by $\delta_y > 0$. For a given $Y=y$, number of PM actions throughout the period L is given by $n_y = \left\lceil r_y(L)-\theta_y/\delta_y \right\rceil$ and $(T_i\,|\,y)=r_y^{-1}(\theta_y+(i-1)\delta_y)$.

Note: This approach extends Yeh and Chang (2007) to a two-dimensional case.

2.5. Availability and Total Maintenance Cost

2.5.1. Availability

The contract period is divided into $n+1$ intervals with the length of the first n and last intervals $L-nT_i$. Suppose that $A(T_i,T_{i+1}\,|\,Y=y)$ shows the availability value at the interval (T_i,T_{i+1}) for $Y=y$. $A(T_i,T_{i+1}\,|\,Y=y)$ is given by:

$$A\left(T_i,T_{i+1}\,|\,Y=y\right)=A_i=\left(T_{i+1}-T_i\right)-t_{pm}-t_{cm}\left(\int_{T_{i-1}}^{T_i} r_i(t\,|\,y)dt\right)/\left(T_{i+1}-T_i\right) \quad (1)$$

where t_{pm} and t_{cm} are the length of time PM and CM action. The formula $A(T_i,T_{i+1}\,|\,Y=y)$ (availability in interval (T_i,T_{i+1})) is used to get the optimal value $T_i, i=1,2,\ldots,n$ that is at least the same as the availability target. Availability during the contract L or the entire interval is described as follows. Suppose $A_S(n,\delta_i \mid y)$ is the expected availability over L conditional on $Y=y$ and it is given by:

$$A_S(n,\delta_i \mid y) = L - t_{pm}n - t_{cm}\left(\sum_{i=1}^{n}\int_{T_{i-1}}^{T_i} r_i(t \mid y)dt + \int_{nT}^{L} r_{n+1}(t \mid y)dt\right)/L \qquad (2)$$

In practice, equipment owners want high availability of equipment to achieve their production targets (e.g. $A_T = 0.940$). In this, we propose an approach to guarantee an availability target of A_S in the entire MSC period by determining the failure rate reduction, $\delta_i, i = 1,2,3,\ldots,n$ such that the availability of each interval is at least equal to the availability target.

2.5.2. Total Maintenance Cost

PM Policy 1: Total maintenance costs include minimal repair costs (including expected downtime penalties) and PM action costs. The total expected cost of a two-dimensional MSC over the period is given by:

$$C_S(n,\delta_i \mid y) = [C_r + C_\tau G(\tau)]\left[\sum_{i=1}^{n}\int_{T_{i-1}}^{T_i} r_i(t \mid y)dt + \int_{nT}^{L} r_{n+1}(t \mid y)dt\right] + n\left(a + b\sum_{i=1}^{n}\delta_i\right) \qquad (3)$$

PM Policy 2: The formula for total maintenance costs consists of minimal repair costs (including expected penalties due to downtime) and PM action costs. The expected total cost for the period L is given by

$$C(n_y,\delta_y,\theta_y) = [C_r + C_\tau \tilde{G}(\tau)]\left[\left(L/\alpha_y\right)^{\beta} + \delta_y\left(\sum_{i=1}^{n}\left((\alpha_y)^{\beta}(\theta_y + (i-1)\delta_y)/\beta\right)^{\beta-1} - L\right)\right] + n(a + b\delta_y) \qquad (4)$$

3. Optimal Decision

This section discusses the optimization scheme (Optimization Schema) of the two-dimensional MSCs under consideration.

3.1. Two-dimensional MSC with PM 1 Policy (Proposed)

Three optimization schemes are considered: Optimization Schemes 1, 2, and 3.

Optimization Scheme 1: Find the value $(n^*, \delta^*_{i|y})$ which maximizes $A_S(n,\delta_i \mid y)$ (given in (2)), and then calculate the total maintenance cost given in (3). Note: Here, the maximization $A_S(n,\delta_i \mid y)$ is done without any constraint on the availability value.

Optimization Scheme 2: Find the value of $(n^*, \delta^*_{i|y})$ which maximizes $A_S(n,\delta_i \mid y)$ (given in (2)) with the availability constraint i.e., $0.940 \le A_T \le 0.999$, and then calculate the total maintenance cost by (3).

Optimization Scheme 3: Find the value of $(n^*, \delta^*_{i|y})$ by considering availability and total cost together. This is done using optimization with two objective functions. Using the Weighted Sum method from the study of Grodzevich and Romanko (2006), the structure of the optimization function with two objectives is given below.

Objective functions:

$$Min\sum_{j=1}^{2} z_j f_j(n,\delta_i \mid y),\ where\ \ f_1(n,\delta_i \mid y) = -A_S(n,\delta_i \mid y)\ and\ \ f_2(n,\delta_i \mid y) = C_S(n,\delta_i \mid y) \qquad (5)$$

Subject to constraints:

(i) $0.940 \le A_T \le 0.999$ (ii) $z_i \ge 0, i = 1,2$ and (iii) $\sum_{i=1}^{2} z_i = 1$

Hence, the value of $(n^*, \delta^*_{i|y})$ obtained will maximize the availability and minimize the total cost simultaneously (not done sequentially as in the previous two schemes).

3.2. Two-dimensional MSC with PM Policy 2

Again, the three optimization schemes are also used to find the optimal solution.

Optimization Scheme 1: (i) First, using the algorithm proposed by Yeh and Chang (2007) finding the optimal value of n_y^*, θ_y^* and δ_y^* which minimizes the expected total cost. (ii) Second, based on the optimal value of (i), then calculate the availability with the equation in (2).

Optimization Scheme 2: In this scheme the value of $\delta_y^*(n)$ must first meet the availability target (A_T) i.e., $0.940 \le A_T \le 0.999$) and then calculate the total cost with the value of $\delta_y^*(n)$ obtained. Thus, Steps 4 and 5 in the algorithm of Yeh and Chang (2007) need to be modified to :

Step 4. Calculate $\delta_y^*(n)$ which maximizes $A(n,\delta_y,\theta_y \mid y)$ and meets the constraint $0.940 \le A_T \le 0.999$. If $\delta_y^*(n) > a\beta / [C_r + C_\tau \tilde{G}(\tau) L - b]$, then calculate $C(n,\delta_y^*(n))$. If not, $(n^*,\delta_y^*,\theta_y^*) = (0,0,0)$ and STOP

Step 5. If $C(n,\delta_y^*(n)) < C(n_1,\delta_y^*(n_1))$ so, set $n_1 = n, (n^*,\delta_y^*) = (n_1,\delta_y^*(n_1)), n = n+1$, and go to step 4. If not, put $n = n+1$ and go to Step 4.

Optimization Scheme 3: Find the value (n^*,δ_y^*) by considering availability and total cost together. The structure of the optimization function with two objectives is given below.

Step 1. Set $n=1$ and $n_1 = 0$.

Step 2. Solve the system of equations with the objective function:

$$Min \sum_{j=1}^{2} z_j f_j(n,\delta_y \mid y) \tag{6}$$

Subject to constraints:

(i) $0.940 \le A_T \le 0.999$ (ii) $z_i \ge 0, i = 1,2$ and (iii) $\sum_{i=1}^{2} z_i = 1$,

where $f_1(n,\delta_y \mid y) = -A(n,\delta_y \mid y)$ is given in (2) and $f_2(n,\delta_y \mid y) = C(n,\delta_y \mid y)$ is given in (4).

To obtain the value $\delta_y^*(n)$, $A(n,\delta_y^*(n))$, and $C(n,\delta_y^*(n))$.

Step 3. If $A(n,\delta_y^*(n)) \ge A(n_1,\delta_y^*(n_1))$ and $C(n,\delta_y^*(n)) < C(n_1,\delta_y^*(n_1))$ set $n_1 = n, (n^*,\delta_y^*) = (n_1,\delta_y^*(n_1)), n = n+1$, and go to Step 2. If not $n_1 = n, (n^*,\delta_y^*) = (n_1,\delta_y^*(n_1))$.

4. Numerical Examples and Discussion

Consider that $F_0(t,\alpha_0)$ is Weibull distribution with $F_0(t,\alpha_0) = 1 - e^{-(t/\alpha_0)^\beta}$, $t \ge 0$, where $\alpha_0 > 0$ is the scale parameter and $\beta > 0$ is the shape parameter. Suppose that each failure incurs costs (including the minimal repair cost and possible penalty cost) and the cost for performing a PM action with maintenance degree δ is $C_p(\delta^*) = a + b\delta_y^*$. A similar form of maintenance cost is used by Supriatna *et al.* (2020). These parameter values were used in Yeh and Chang (2007) and will be considered in this section given in Table 1.

Table 1 Parameter value considered

Parameter	α_0	ρ	β	L	t_{pm}	t_{cm}
Value	7, 10, 13	1.2	1.5	3*, 10*, 20*	2.16	4.32
Unit	Months	-	-	months	hours	hours

*) Values are taken from Yeh and Chang (2007).

In this section, we shall evaluate the performance of two-dimensional MSC with PM Policy 1 (proposed PM) and two-dimensional with PM Policy 2.

4.1. Discussion of MSC 2D results with PM 1

Tables 2 and 3 show the results for low usage (y=0.9), comparing results with Schemes 1 and 2 and then results with Schemes 2 and 3. From Tables 2 and 3, we have the following findings. Increasing the parameter value of α_0 gives the same or increased availability for Schemes 1-3. This is expected because larger α_0 means higher reliability (this agrees with the result of Yeh and Chang (2007) for a 1D MSC case). The results with Scheme 2 always give higher availability values than Scheme 1. In addition, Scheme 2 also provides a lower total cost of up to 71.70% (see Table 2, for the parameter $(\alpha_0, L) = (7,3)$) than the results of Scheme 1. The total cost is smaller because the PM Policy 1 improves reliability by optimizing δ (is not constant). Furthermore, Scheme 3 also provides an availability value that is as large as the results of Scheme 2. The advantage of Scheme 3 is that the total cost obtained is lower than the total cost with Schemes 1 and 2. Total maintenance costs can be reduced by up to 71.70% (see Table 2 on parameters $(\alpha_0, L) = (7,3)$). Thus, the best optimization scheme for PM 1 policy is Scheme 3, and this pattern also holds for medium and high usage rates (Note: the results cannot be included due to the number of page limitations of a paper).

Table 2 Results of MSC 2D with PM 1 using Schemes 1 and 2 for low usage (y=0.9)

		Scheme 1			Scheme 2			Δ	
α_0	L	(n*, δ*)	As (n*, δ*)	Cs (n*, δ*)	(n*, δ*)	As (n*, δ*)	Cs (n*, δ*)	A	C
7	3	(**1**, 0.12)	0.995	20.92	(**1**, 0.22)	0.996	5.92	0.10	**-71.70**
	10	(**2**, 0.30)	0.998	16.40	(**2**, 0.16;0.46)	0.999	9.74	0.10	-40.61
	20	(**3**, 0.38)	0.998	28.79	(**2**,0.32;0.54)	0.999	20.46	0.10	-28.93
10	3	(**1**, 0.13)	0.996	5.36	(**1**, 0.13)	0.996	5.36	0.00	0.00
	10	(**3**, 0.16)	0.997	15.88	(**3**,0.03;0.08;0.37)	0.999	15.88	0.20	0.00
	20	(**6**, 0.20)	0.996	29.05	(**3**, 0.16;...;0.35)	0.999	21.90	0.30	-24.61

Table 3 Results of MSC 2D with PM 1 using Schemes 2 and 3 for low usage (y=0.9)

		Scheme 2			Scheme 3			Δ	
α_0	L	(n*, δ*)	As (n*, δ*)	Cs(n*, δ*)	(n*, δ*)	As (n*, δ*)	Cs(n*, δ*)	A	C
7	3	(**1**, 0.22)	0.996	5.92	(**1**, 0.22)	0.996	5.92	0.00	0.00
	10	(**2**, 0.16;0.46)	0.999	9.74	(2, 0.17;0.45)	0.999	9.74	0.00	0.00
	20	(**2**,0.32;0.54)	0.999	20.46	(**2**,0.35;0.52)	0.999	13.79	0.00	-32.60
10	3	(**1**, 0.13)	0.996	5.36	(**1**, 0.13)	0.996	5.36	0.00	0.00
	10	(**3**,0.03;0.08;0.37)	0.999	15.88	(**3**,0.02;0.08;0.38)	0.999	15.88	0.00	0.00
	20	(**3**, 0.14;...;0.36)	0.999	36.90	(**3**,0.17;...;0.34)	0.999	16.90	0.00	-54.20

4.2. Discussion of MSC 2D results with PM 2

We have the following findings from Table 4. The results of Schemes 1 and 2 are not different. Scheme 3, which optimizes two performance measures simultaneously, gives the best results. Availability using Scheme 3 increases up to 0.10%. In addition, the total cost of maintenance can also be reduced by up to 43.81% (see Table 4, in parameter $(\alpha_0, L) = (7,20)$. The Scheme 3 is also the best for availability and total cost measures for medium and high usages.

Table 4 Results of MSC 2 D with PM 2 Schemes 1, 2 and 3 with a low level of use (*y*=0.9)

		Schemes 1 and 2			Scheme 3			Δ	
α_0	L	(n*, δ*)	A (n*, δ*)	C (n*, δ*)	(n*, δ*)	A (n*, δ*)	C (n*, δ*)	A	C
7	3	(**1**, 0.12)	0.995	20.94	(**1**, 0.10)	0.995	19.35	0.00	-7.59
	10	(**2**, 0.15)	0.996	92.45	(**2**, 0.17)	0.997	65.83	0.10	-28.79
	20	(**3**, 0.16)	0.996	237.03	(**3**, 0.11)	0.997	133.19	0.10	**-43.81**
10	3	(**1**, 0.06)	0.996	13.43	(**1**, 0.06)	0.996	13.43	0.00	0.00
	10	(**3**, 0.06)	0.996	47.86	(**3**, 0.04)	0.996	43.13	0.00	-9.88
	20	(**7**, 0.04)	0.996	89.43	(**7**, 0.03)	0.996	71.88	0.00	-19.62

4.3. Comparison between the two MSC 2D

In each of the 2D MSCs considered, Scheme 3 gives the best results. MSC 2D with PM 1 is superior both in terms of performance measures of availability and total cost (see Table 5). Regarding availability, MSC 2D with PM 1 is up to 0.40% superior (see Table 5, in parameter $(\alpha_0, L) = (7,10)$) to MSC 2D with PM Policy 2. Meanwhile, regarding total costs, MSC 2D with PM 1 provides a smaller total cost of almost 1/4 times the 2D MSC with PM 2 - see Table 8 with the 2D MSC parameter $(\alpha_0, L) = (7, 20)$ with PM 2 up to 396.38%.

The advantage of the PM 1 policy is the optimization of δ (the reduction in intensity function) for each PM action. Whereas in the PM 2 policy, the value δ for each PM action is the same, even though the longer the age of the equipment or the higher the usage rate. This advantage makes PM Policy 1 (proposed PM) superior to PM Policy 2. For an agent who offers a 2D MSC with an availability target, the results of this study are helpful in determining maintenance policies that can meet availability targets with minimum total maintenance cost.

Table 5 Results of MSC 2D with PM 1 and PM 2 Policies, moderate use (y=1.0)

		PM 1 Policy			PM 2 Policy			Δ *	
α_0	L	(n*, δ*)	A (n*, δ*)	C (n*, δ*)	(n*, δ*)	A (n*, δ*)	C (n*, δ*)	A	C
7	3	(**1**, 0.18)	0.996	6.06	(**1**, 0.08)	0.996	16.86	0.00	178.22
	10	(**3**, 0.11;0.11;0.46)	0.999	15.75	(**4**, 0.05)	0.995	47.91	**0.40**	204.19
	20	(**3**, 0.25;0.25;0.46)	0.999	17.94	(**5**, 0.06)	0.997	89.05	-0.20	**396.38**
10	3	(**1**, 0.11)	0.996	4.93	(**1**, 0.04)	0.996	12.27	0.00	148.88
	10	(**3**, 0.03;0.03;0.34)	0.999	15.00	(**3**, 0.04)	0.996	34.82	-0.30	132.13
	20	(3, 0.13;0.13;0.30)	0.999	17.84	(**3**, 0.05)	0.998	75.62	-0.10	323.88

Δ *: the difference between the performance of PM 2 and PM 1 where A (availability) and C (cost); a positive sign means PM 1 < PM 2, otherwise negative means PM 1 > PM 2.

5. Conclusions

In this paper, a two-dimensional Maintenance Service Contract (MSC) study is conducted with (1) periodic and (2) non-periodic PM policies by considering availability performance measures and total maintenance costs. The two-dimensional MSC with periodic PM policy provides the best performance – in terms of availability (can guarantee an availability target) and low total costs at any rate of usage considered. Here, a two-dimensional MSC study was conducted from the agent's point of view. One of the further research topics is a two-dimensional MSC study that takes into account the two parties- i.e., the agent and the consumer, who are very concerned about the price of MSC. This topic can be modeled by a game theory formulation and provides optimal results for both parties. In addition, from an equipment maintenance perspective, MSC can also consider condition-based preventive maintenance options so equipment performance can be even better.

Furthermore, as the maintenance services can be provided by the Original Equipment Manufacturer (OEM) or the agent, another interesting topic is to study a 2D MSC involving three parties– i.e., the Original Equipment Manufacturer (OEM), the agent, and the consumer. The integration of three-party decision problems is an interesting topic because it can mathematically describe the relationship of competition or cooperation between agents. The formulation can be done using two-level game theory and it is expected to choose the optimal option for consumers as well as the optimal three-party coordinated option. The findings will be very beneficial for the parties involved with the following details: (i) the agent can achieve the availability target with minimum total maintenance costs, and (ii) the consumer benefits from the optimal target of equipment availability and MSC price to maximize profit. Research on these topics is on the way.

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